Infinite elements for exterior Helmholtz resonance problems based on a frequency dependent complex scaling

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Complex scaling is a popular method to treat scattering and resonance problems in open domains. Thereby the unbounded domain is decomposed into a bounded interior and an unbounded exterior part. Subsequently the technique of complex scaling is applied to the exterior domain to obtain exponentially decreasing solutions. Finally the complex scaled exterior is usually truncated and discretized using finite elements.

In our work we suggest a number of improvements to the method described above. To resonance problems usually frequency independent scalings are applied, to conserve the linearity of the resulting eigenvalue problem. Unfortunately, a frequency independent complex scaling works well only for a very narrow range of frequencies and the quality of the approximation depends heavily on the specific choice of the scaling function. To overcome this problem we use frequency dependent scaling functions, as it is common when treating scattering problems, for resonance problems as well. This approach leads to polynomial or rational eigenvalue problems instead of linear ones (cf. [2]).

For discretizing the exterior complex scaled problems we use a tensor product method, describing the exterior by a normal and an interface coordinate. Due to this ansatz we evade having to explicitly mesh the exterior domain. To avoid truncation and obtain super-algebraic approximation properties we make use of infinite elements in normal direction, which are based on Hardy space infinite Elements (cf. [1]).

We solve the resulting discrete eigenvalue problems by making use of an adapted version of the shift-and-invert Arnoldi algorithm. Applying this method requires no significant extra computational effort compared to solving linear eigenvalue problems.

References.

[1] T. Hohage, L. Nannen, Hardy space infinite elements for scattering and resonance problems, *SIAM J. Numer. Anal.* **47** (2009), pp. 972–996.

[2] L. Nannen, M. Wess, Computing scattering resonances using perfectly matched layers with frequency dependent scaling functions, *BIT* **58** (2018), pp. 373–395.